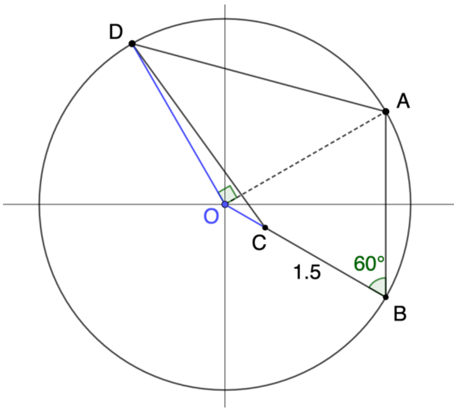
Assessment Schedule – 2025

Scholarship Calculus (93202)

Evidence Statement

Q	Solution
ONE (a)	$\sqrt[3]{125^{x^3-5x^2+11x-3}} = \sqrt[4]{5^{4x^2+12}}$ $\left[(5^3)^{x^3-5x^2+11x-3}\right]^{\frac{1}{3}} = \left[5^{4x^2+12}\right]^{\frac{1}{4}}$ $5^{x^3-5x^2+11x-3} = 5^{x^2+3}$ $x^3-5x^2+11x-3 = x^2+3$ $x^3-6x^2+11x-6=0$ $(x-1)(x-2)(x-3)=0$ $x = 1, 2,$ and 3
(b)	$z = 2 - \frac{2+\mathrm{i}}{\mathrm{i}z}$ $\mathrm{i}z^2 = 2\mathrm{i}z - 2 - \mathrm{i}$ $\mathrm{i}z^2 - 2\mathrm{i}z + 2 + \mathrm{i} = 0$ $z^2 - 2z - 2\mathrm{i} + 1 = 0$ $(z-1)^2 = 2\mathrm{i}$ $z = 1 \pm (1+\mathrm{i})$ $z = -\mathrm{i}$ and $2+\mathrm{i}$
(c)	$z^{2025} = \bar{z}$ Firstly, note that $z = 0$ is a solution. Now, multiply both sides of the equation by z, giving $z^{2026} = z\bar{z} = r^2$. This equation has 2026 non-zero complex solutions, so, the original equation has a total of 2027 solutions.
(d)	From the information we can contruct the diagram shown. $\mathrm{OA} = \mathrm{OB} = \mathrm{OD} = r$ $\mathrm{OC} = r - 1.5 = \frac{1}{r}$ $2r^2 - 3r - 2 = 0$ $(2r+1)(r-2) = 0$ $r = 2$ is the only valid solution. Area $\triangle\mathrm{AOB} = \frac{1}{2} \times 2^2 \times \sin 60° = \sqrt{3}$ Area $\triangle\mathrm{AOD} = \frac{1}{2} \times 2^2 = 2$ Area $\triangle\mathrm{COD} = \frac{1}{2} \times \frac{1}{2} \times 2 \times \sin 150° = \frac{1}{4}$ So, the area of quadrilateral $ABCD$ is $\sqrt{3} + 2 - \frac{1}{4} = \frac{7}{4} + \sqrt{3}$. 

Scholarship level	**Outstanding level**
Q1(a) • **1s mark** for simplifying to Line 3 • **1s mark** for all three correct x-values Q1(b) • **1s mark** for Line 4 quadratic and square root of 2i found • **1s mark** for both correct z-values Q1(c) • **1s mark** for recognising z^{2026} = constant • **1s mark** for stating 2026 solutions with explanation Q1(d) • **1s mark** for diagram with B, C, and D in right place • **1s mark** for correct value of r *8 marks available* **Max 6 marks awarded**	Q1(c) • **1o mark** for correct number of solutions Q1(d) • **1o mark** for correct area of quadrilateral

TWO (a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 2 + 2t \quad \frac{\mathrm{d}y}{\mathrm{d}t} = p'(t)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{p'(t)}{2+2t}$ $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{p''(t)(2+2t) - 2p'(t)}{(2+2t)^2} \times \frac{1}{2+2t} = \frac{3}{4(1+t)}$ $p''(t)(2+2t) - 2p'(t) = \frac{3(2+2t)^3}{4(1+t)}$ $p''(t)(1+t) - p'(t) = 3(1+t)^2$ Now let $p(t)$ be a cubic: $p(t) = at^3 + bt^2 + ct + d$ $p'(t) = 3at^2 + 2bt + c$ $p''(t) = 6at + 2b$ Substituting these into the equation: $(6at + 2b)(1+t) - (3at^2 + 2bt + c) = 3(1+t)^2$ $3at^2 + 6at + 2b - c = 3t^2 + 6t + 3$ Equating coefficients: $a = 1$ $2b - c = 3$ Using the given information: $p'(1) = 3 + 2b + c = 6$ $2b + c = 3$ $2b - c = 3$ $b = 1.5, c = 0$ $p(1) = 1 + \frac{3}{2} + d = 2.5$ $d = 0$ So the polynomial is $p(t) = t^3 + 1.5t^2$ and $p(2) = 14$.

(b)	As f is defined for all real numbers and $f(-x) = -f(x)$, $f(0) = 0$. $f(1-x) = f(1+x)$ $x = 0$ gives $f(1) = f(1) = 2025$ $x = 1$ gives $f(0) = f(2) = -f(-2) = 0$ $x = 2$ gives $f(-1) = f(3) = -f(1) = -2025$ $x = 3$ gives $f(-2) = f(4) = -f(2) = 0$ $x = 4$ gives $f(-3) = f(5) = -f(3) = 2025$ $x = 5$ gives $f(-4) = f(6) = -f(4) = 0$ $x = 6$ gives $f(-5) = f(7) = -f(5) = -2025$ etc. 2024 is a multiple of 4, so $f(1) + f(2) + \dots + f(2024) = 0$ Hence the required sum $f(1) + f(2) + \dots + f(2025) = 2025$.
(c)	$g(x) = f(x) - x^2 = ax^3 + bx^2 + cx + d - x^2 = ax^3 + (b-1)x^2 + cx + d$ Note that this function has roots at 2, 3, and 4. $g(x) = a(x-2)(x-3)(x-4)$ $f(x) = a(x-2)(x-3)(x-4) + x^2$ Line AB has equation $y = 5x - 6$ Line AC has equation $y = 6x - 8$ Line BC has equation $y = 7x - 12$ If we equate $f(x)$ with each line, we can get the x-values of D, E, and F respectively. $a(x-2)(x-3)(x-4) + x^2 = 5x - 6$ has solutions of 2, 3, and x_{D} $a(x-2)(x-3)(x-4) + (x-2)(x-3) = 0$ Dividing out the common factors to find x_{D} gives: $a(x-4) + 1 = 0$ $x_{\mathrm{D}} = 4 - \frac{1}{a}$ Similarly: $x_{\mathrm{E}} = 3 - \frac{1}{a}$ $x_{\mathrm{F}} = 2 - \frac{1}{a}$ $4 - \frac{1}{a} + 3 - \frac{1}{a} + 2 - \frac{1}{a} = 25$ $a = -\frac{3}{16}$ $f(0) = -\frac{3}{16}(-2)(-3)(-4) = 4.5$

Scholarship level	**Outstanding level**
Q2(a) • **1s mark** for correct 2nd derivative in terms of $p(t)$ etc. • **1s mark** for Line 5 expression • **1s mark** for quadratic equation and value of a • **1s mark** for values of b, c, and d, and the value of $p(2)$ Q2(b) • **1s mark** for recognising $f(0) = f(2) = 0$ • **1s mark** for finding $f(5) = 2025$ Q2(c) • **1s mark** for factorising $g(x)$ using the three roots • **1s mark** for finding x_{D} in terms of a or equivalent *8 marks available* **Max 6 marks awarded**	Q2(b) • **1o mark** for the correct answer with working Q2(c) • **1o mark** for the correct value of a and $f(0)$

THREE (a)	$\text{LHS} = \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} = \frac{1-\sin x}{1-\sin^2 x} = \frac{1-\sin x}{\cos^2 x} = \sec^2 x - \sec x \tan x = \text{RHS}$ $\therefore \int_0^{\frac{2\pi}{3}} \frac{1}{1+\sin x}\,\mathrm{d}x = \int_0^{\frac{2\pi}{3}} (\sec^2 x - \sec x \tan x)\,\mathrm{d}x$ $= [\tan x - \sec x]_0^{\frac{2\pi}{3}} = \tan\left(\frac{2\pi}{3}\right) - \sec\left(\frac{2\pi}{3}\right) + 1 = 3 - \sqrt{3}$
(b)(i)	$\tan\left(\frac{3\pi}{8}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$ $= \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{8}}{1 - \left(\tan\frac{\pi}{4}\tan\frac{\pi}{8}\right)}$ $= \frac{1+\sqrt{2}-1}{1-(\sqrt{2}-1)}$ $= \frac{\sqrt{2}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}}$ $= \frac{2+2\sqrt{2}}{2} = 1+\sqrt{2}$

(ii)	Let $u = \tan\left(\frac{x}{2}\right)$ and $v = \tan y$. From the equations and the compound angle formula we get: $uv = \sqrt{2} - 1$ $\frac{u+v}{1-uv} = 1 + \sqrt{2}$ Solving simultaneously: $u + v = \left(1+\sqrt{2}\right)\left(1-\sqrt{2}+1\right)$ $u + v = \sqrt{2}$ $u\left(\sqrt{2} - u\right) = \sqrt{2} - 1$ $u^2 - \sqrt{2}\,u + \left(\sqrt{2} - 1\right) = 0$ $u = \tan\left(\frac{x}{2}\right) = \frac{\sqrt{2} \pm \sqrt{2 - 4\left(\sqrt{2}-1\right)}}{2}$ The negative root gives: $\tan\left(\frac{x}{2}\right) = \sqrt{2} - 1 = \tan\left(\frac{\pi}{8}\right)$ $\frac{x}{2} = n\pi + \frac{\pi}{8}$ $x = 2n\pi + \frac{\pi}{4}$ $\tan\left(n\pi + \frac{\pi}{8} + y\right) = 1 + \sqrt{2} = \tan\left(\frac{3\pi}{8}\right)$ $n\pi + \frac{\pi}{8} + y = k\pi + \frac{3\pi}{8}$ $y = \frac{\pi}{4} + m\pi$ The positive root gives: $\tan\left(\frac{x}{2}\right) = 1 = \tan\left(\frac{\pi}{4}\right)$ $\frac{x}{2} = n\pi + \frac{\pi}{4}$ $x = 2n\pi + \frac{\pi}{2}$ $\tan\left(n\pi + \frac{\pi}{4} + y\right) = 1 + \sqrt{2} = \tan\left(\frac{3\pi}{8}\right)$ $n\pi + \frac{\pi}{4} + y = k\pi + \frac{3\pi}{8}$ $y = \frac{\pi}{8} + m\pi$ So, all solutions are given by $(x, y) = \left(2n\pi + \frac{\pi}{4}, m\pi + \frac{\pi}{4}\right)$ or $(x, y) = \left(2n\pi + \frac{\pi}{2}, m\pi + \frac{\pi}{8}\right)$.

(c)	Let C be the cost of running the cable. Assuming a straight cable, there are three possibilities: 1. Running the cable from A directly to the mainland and then along the coast to B, giving $C = 30W + 40L$. 2. Running the cable directly to B, giving $C = 50W$. 3. Running the cable to some point x km further along the coast, where $0 < x < 40$, then along the coast to B. $C = W\sqrt{30^2 + x^2} + L(40 - x)$ $\frac{\mathrm{d}C}{\mathrm{d}x} = \frac{Wx}{\sqrt{30^2 + x^2}} - L = 0$ $Wx = L\sqrt{900 + x^2}$ $W^2x^2 = L^2(900 + x^2)$ $(W^2 - L^2)x^2 = 900L^2$ $x = \sqrt{\frac{900L^2}{W^2 - L^2}} = \frac{30L}{\sqrt{W^2 - L^2}}$ Note that this local minimum only exists if $W > L > 0$, which we are told is true! **Scenario One:** If $W = 1.5$ L Then $x \approx 26.83$ I.e. run the cable to a point on the mainland 13.17 km from point B. **Scenario Two:** If $W = 1.2$ L Then $x \approx 45.23$ This occurs past point B. Because the cost to run the cable under the water is only slightly more expensive than on the mainland, it is cheaper to run the cable directly from A to B.

Scholarship level	Outstanding level
Q3(a) • **1s mark** for proof of the identity • **1s mark** for correct answer with integration Q3(b)(i) • **1s mark** for correct use of the compound angle formula • **1s mark** for finding the exact value for tan(3π/8) Q3(b)(ii) • **1s mark** for one general solution for x Q3(c) • **1s mark** for the cost expression and derivative • **1s mark** for finding the x value of the local minimum • **1s mark** for correct answer for one case *8 marks available* **Max 6** marks awarded	Q3(b)(ii) • **1o mark** for both general solutions correct for x and y Q3(c) • **1o mark** for complete solution considering both cases

FOUR (a)(i)	Using the symmetry of the diagram: $\int_0^1 \sin^{-1}(x)\,\mathrm{d}x = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \cos x\,\mathrm{d}x$ $= \frac{\pi}{2} - [\sin x]_0^{\frac{\pi}{2}}$ $= \frac{\pi}{2} - 1$ OR Integrating upwards, finding the area between the function and the y-axis: $\int_0^{\frac{\pi}{2}} x\,\mathrm{d}y = \int_0^{\frac{\pi}{2}} \sin y\,\mathrm{d}y = -[\cos y]_0^{\frac{\pi}{2}} = 1$ The area below the function is found by taking the area above away from a rectangle. $\int_0^1 \sin^{-1}(x)\,\mathrm{d}x = \frac{\pi}{2} - 1$ OR Using integration by parts. This method was not expected, but can be used by looking ahead to part (ii): $\int_0^1 \sin^{-1}(x) \times 1\,\mathrm{d}x = [x\sin^{-1}(x)]_0^1 - \int_0^1 \frac{1}{\sqrt{1-x^2}} \times x\,\mathrm{d}x$ $= \frac{\pi}{2} + \frac{1}{2}\int_0^1 \frac{-2x}{\sqrt{1-x^2}}\,\mathrm{d}x$ $= \frac{\pi}{2} + \frac{1}{2}[2\sqrt{1-x^2}]_0^1\,\mathrm{d}x$ $= \frac{\pi}{2} - 1$
(ii)	$x = \sin y$ $\frac{\mathrm{d}x}{\mathrm{d}y} = \cos y$ Now using a trig identity (alternatively, a right-angled triangle) $\sin^2 y = x^2$ $1 - \cos^2 y = x^2$ $\cos y = \sqrt{1-x^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1-x^2}}$ $g(x) = \sin^{-1}(2x) + \sin^{-1}\left(\frac{\pi}{4} - 2x\right)$ $g'(x) = \frac{2}{\sqrt{1-(2x)^2}} - \frac{2}{\sqrt{1-\left(\frac{\pi}{4}-2x\right)^2}} = 0$ $\frac{\pi}{4} - 2x = \pm 2x$ $x = \frac{\pi}{16}$ The minimum value of $g(x)$ is: $g\left(\frac{\pi}{16}\right) = 2\sin^{-1}\left(\frac{\pi}{8}\right) \approx 0.8071$

(b)

Using the note:

$$f(f'(x)) = f\left(f^{-1}(x)\right) = x$$

Now substituting the solution $f(x) = \left(\frac{1}{r^r}\right)^{\frac{1}{r+1}} x^r$ into the differential equation:

$$f(f'(x)) = \left(\frac{1}{r^r}\right)^{\frac{1}{r+1}} \left(\left(\frac{1}{r^r}\right)^{\frac{1}{r+1}} \times rx^{r-1}\right)^r = x$$

$$\left(\frac{1}{r^r}\right)^{\frac{1}{r+1}} \left(\left(\frac{1}{r^r}\right)^{\frac{r}{r+1}} \times r^r x^{r(r-1)}\right) = x$$

$$\left(\frac{1}{r^r}\right)^{\frac{r+1}{r+1}} \times r^r x^{r(r-1)} = x$$

$$x^{r(r-1)} = x$$

$$r(r-1) = 1$$

$$r^2 - r - 1 = 0$$

Taking the positive root:

$$r = \frac{1+\sqrt{5}}{2}$$

Note: candidates may find the inverse function and substitute into the RHS of the DE.

(c)(i)

$\lim\limits_{x \to -\infty} h(x) = 0$ as $e^x \to 0$, so $\sin\left(e^x\right) \to 0$

$\lim\limits_{x \to \infty} h(x)$ does not exist because $h(x)$ never settles down.

As $e^x \to \infty$, $\sin\left(e^x\right)$ oscillates between -1 and 1.

(ii)

Let the antiderivative of $\sin\left(e^x\right) = F(x)$.

$$I = \int_a^{a+1} \sin\left(e^x\right) \mathrm{d}x = [F(x)]_a^{a+1} = F(a+1) - F(a)$$

To find the maximum value, we differentiate:

$$\frac{\mathrm{d}I}{\mathrm{d}a} = F'(a+1) - F'(a) = \sin\left(e^{a+1}\right) - \sin\left(e^a\right) = 0$$

$$\sin\left(e^{a+1}\right) = \sin\left(e^a\right)$$

$$e^{a+1} = n\pi + (-1)^n e^a$$

$$e^a\left(e - (-1)^n\right) = n\pi$$

$$a = \ln\left(\frac{n\pi}{e - (-1)^n}\right)$$

a is only defined for $n > 0$.

n	Exact a	Approximate a
1	$\ln\left(\frac{\pi}{e+1}\right)$	–0.1685
2	$\ln\left(\frac{2\pi}{e-1}\right)$	1.2988
3	$\ln\left(\frac{3\pi}{e+1}\right)$	0.9301
4	$\ln\left(\frac{4\pi}{e-1}\right)$	1.9897

Using the graph, it appears that the first value would give the biggest area below the curve and above the x-axis, so the integral is maximised when $a = \ln\left(\frac{\pi}{e+1}\right)$.

The other values of a lie close to a root of the function and $a + 1$ on the other side, so these values are expected to give minimum values for the integral, with area above and below cancelling each other.

Scholarship level	**Outstanding level**
Q4(a)(i) • **1s mark** for rewriting integral (Line 1 in any method) • **1s mark** for correct answer with integration shown Q4(a)(ii) • **1s mark** for correct proof • **1s mark** for correct answer with differentiation shown Q4(b) • **1s mark** for substitution into the DE (Line 4) Q4(c)(i) • **1s mark** for correct explanation for both limits Q4(c)(ii) • **1s mark** for correct derivative (Line 2 of working) • **1s mark** for correct expression for a in terms of n *8 marks available* **Max 6 marks awarded**	Q4(b) • **1o mark** for correct value of r with working Q4(c)(ii) • **1o mark** for correct value of a with explanation

Sufficiency Statement

Score 1–4, no award	Score 5–6, Scholarship	Score 7–8, Outstanding Scholarship
Shows understanding of relevant mathematical concepts, and some progress towards solutions to problems.	Application of high-level mathematical knowledge and skills, leading to partial solutions to complex problems.	Application of high-level mathematical knowledge and skills, perception, and insight / convincing communication shown in finding correct solutions to complex problems.

Cut Scores

Scholarship	Outstanding Scholarship
17–26	27–32